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# Multi-phases in gauge theories on non-simply connected spaces

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## Abstract

It is pointed out that phase structures of gauge theories compactified on non-simply connected spaces are not trivial. As a demonstration, an  $SU(2)$  gauge model on  $M^3 \otimes S^1$  is studied, and it is shown to possess three phases: Hosotani, Higgs and coexisting phases. The critical radius and the order of the phase transitions are determined explicitly. A general discussion about phase structures for small and large scales of compactified spaces is given. The appearance of phase transitions suggests a GUT scenario in which the gauge hierarchy problem is replaced by the dynamical problem of the stabilization of the radius of a compactified space in the vicinity of a critical radius.

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# 1 INTRODUCTION

Recently, it has been discovered that field theories with nontrivial backgrounds on extra dimensions have rich phase structures. Magnetic flux passing through a circle  $S^1$  can cause spontaneous breakdown of the translational invariance of  $S^1$ [1]. A kink-like configuration is then generated dynamically as a vacuum configuration. A second-order phase transition occurs at some critical radius of  $S^1$ , and the translational invariance is restored below this critical radius. The appearance of critical radii is one of the characteristic features of such models. The existence of magnetic flux influences the spectra of models and causes nonstandard patterns of symmetry breaking[2]. Spontaneous breaking of translational invariance naturally leads to a new mechanism of supersymmetry breaking[3], because translations and supersymmetry transformations are related by the supersymmetry algebra.

The magnetic flux background on one extra dimension  $S^1$  can be extended to higher extra dimensions. In Ref.[4], a monopole background on a sphere  $S^2$  is shown to cause spontaneous breakdown of the rotational invariance of  $S^2$ . Vortex configurations are dynamically generated as vacuum configurations, and the number of vortices is found to be proportional to the magnetic charge of the monopole. A second-order phase transition occurs at some critical radius of  $S^2$ , and the rotational symmetry turns out to be restored below the critical radius.

In this paper, we point out that multi-phases can appear in gauge theories on non-simply connected spaces even without any background field configurations. Studies of gauge theories, for instance, on  $M^D \otimes S^1$  (where  $M^D$  denotes a  $D$ -dimensional Minkowski space-time) have a long history and have been made from various points of view[5]-[10]. Nevertheless, as far as we know, phase structures of gauge theories with Higgs fields on non-simply connected spaces have not been investigated, and a variety of phase structures of such gauge theories have been overlooked so far. Since the full analysis of such theories is beyond the scope of this letter, we restrict our consideration to a simple  $SU(2)$  gauge model on  $M^3 \otimes S^1$  as a demonstration. This model turns out to possess the three phases displayed in Fig. 1. The solid curve and dashed curves denote the critical lines of the first- and second-order phase transitions, respectively. A nontrivial Wilson line is generated in the Hosotani phase, and the Higgs field acquires a nonvanishing vacuum expectation value in the Higgs phase. The coexisting phase is a hybrid of these two phases.

The paper is organized as follows. In the next section, an  $SU(2)$  gauge model on  $M^3 \otimes S^1$  is introduced. In Section 3, the phase structure of this model is clarified. In Section 4, as a phenomenological application of our results, a GUT scenario is proposed, and the manner in which the hierarchy problem is reinterpreted in this scenario is discussed. Section 5 is devoted to conclusions and discussion.

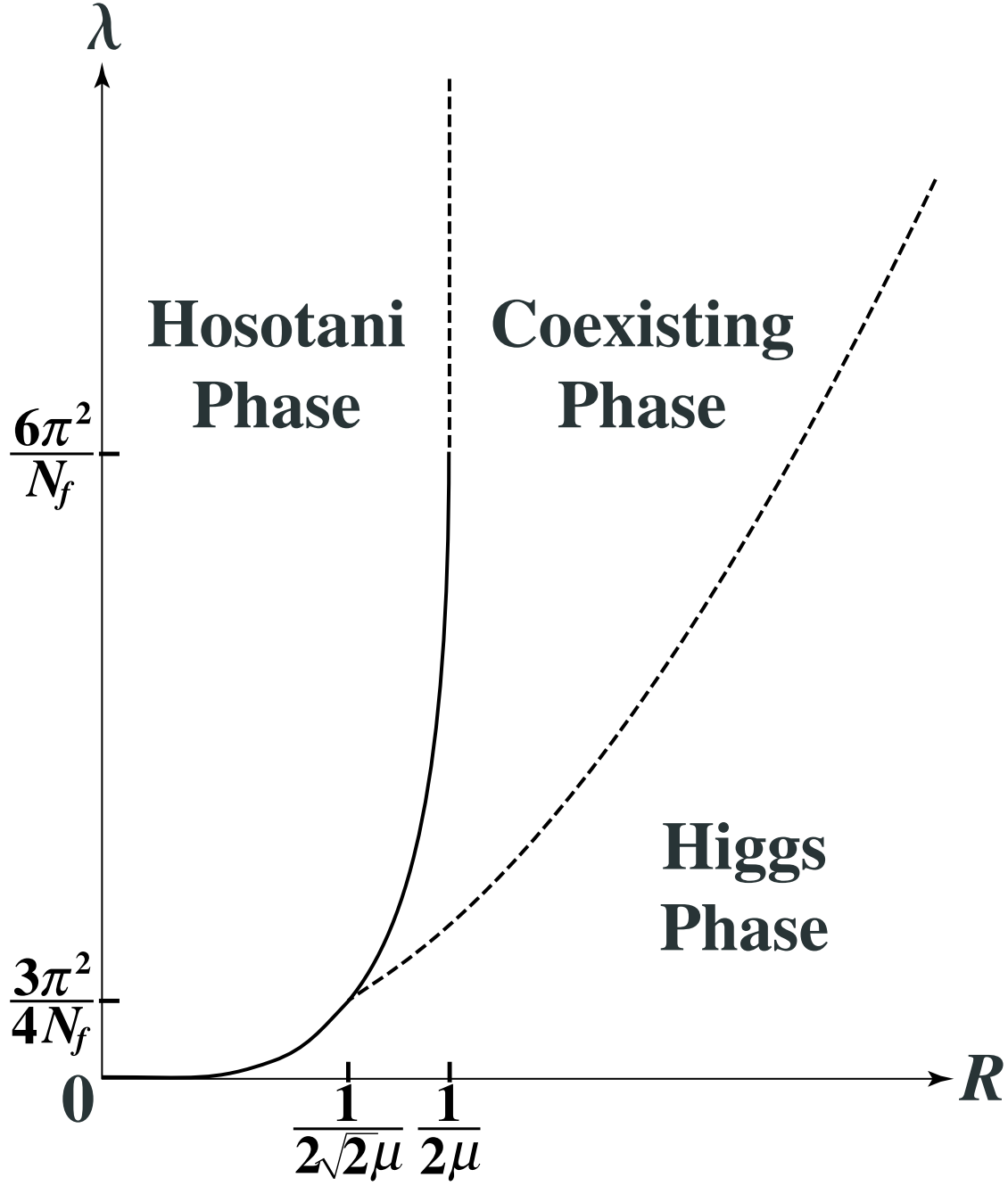


Figure 1: Phase diagram of the  $SU(2)$  gauge model on  $M^3 \otimes S^1$ . The solid curve and dashed curves denote the first- and second-order phase transitions, respectively. The values  $R$  and  $\lambda$  are the radius of  $S^1$  and the Higgs coupling, respectively.

## 2 $SU(2)$ Model

In order to demonstrate that gauge theories compactified on non-simply connected spaces are not trivial, we investigate an  $SU(2)$  gauge model on  $M^3 \otimes S^1$  with  $N_f$  massless fermions and a Higgs boson in the fundamental representation of  $SU(2)$ . Here,  $M^3$  denotes a three-dimensional Minkowski space-time, and  $S^1$  is a circle of a radius  $R$ . The action we consider is

$$S = \int d^3x \int_0^{2\pi R} dy \left\{ -\frac{1}{2} \text{tr} F_{MN} F^{MN} + \sum_{I=1}^{N_f} \bar{\psi}_I \gamma^M D_M \psi_I + (D_M \phi)^\dagger D^M \phi - V(\phi) \right\}, \quad (1)$$

where  $D_M = \partial_M + igA_M$  is a covariant derivative and

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2. \quad (2)$$

The indices  $M$  and  $N$  run from 0 to 3, and  $x^\mu$  ( $\mu = 0, 1, 2$ ) and  $y$  are the coordinates on  $M^3$  and  $S^1$ , respectively. All the fields are assumed to obey periodic boundary conditions in the  $S^1$  direction.<sup>5</sup> To investigate the vacuum configuration, we take the vacuum expectation values of the bosonic fields to be of the forms

$$\langle A_\mu \rangle = 0, \quad (\mu = 0, 1, 2), \quad (3)$$

$$\langle A_y \rangle = \frac{1}{gR} \begin{pmatrix} \alpha & 0 \\ 0 & -\alpha \end{pmatrix}, \quad (4)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad (5)$$

with real positive  $v$ .

The leading order correction to the effective potential for  $\alpha$  comes from the fermion one-loop diagram and is given by [8]

$$\begin{aligned} \Delta V(\alpha; R) &= \frac{N_f}{2\pi^6 R^4} \sum_{n=1}^{\infty} \frac{\cos(2\pi n \alpha)}{n^4} \\ &= -\frac{N_f}{6\pi^2 R^4} \left\{ \alpha^4 - 2\alpha^3 + \alpha^2 - \frac{1}{30} \right\}, \end{aligned} \quad (6)$$

where the last equality holds only for  $0 \leq \alpha \leq 1$ . Although there are other one-loop corrections coming from the gauge, ghost and Higgs fields, we ignore them in the following analysis to avoid unnecessary complexity. This simplification may be justified by taking  $N_f$  to be sufficiently large. The effective potential for  $\alpha$  and  $v$  is then given by

$$V_{\text{eff}}(\alpha, v; R) = -\frac{\mu^2}{2} v^2 + \frac{\lambda}{8} v^4 + \frac{\alpha^2}{2R^2} v^2 + \Delta V(\alpha; R). \quad (7)$$

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<sup>5</sup> Since  $S^1$  is multiply connected, we could impose twisted boundary conditions on the fields,[5] but we are not interested in phase transitions caused by such boundary effects[1, 2, 3] in this paper.

Note that the third term comes from the covariant derivative of the Higgs field, which gives the interaction term between the gauge and the Higgs fields. The interaction term turns out to be crucial to determine the phase structure of gauge theories with Higgs fields on multiply connected spaces. In the next section, we determine the vacuum configuration that minimizes the effective potential (7) and clarify the phase structure of the model.

### 3 Phase Structure

To find the vacuum configuration, we examine the extremum conditions of the effective potential,

$$0 = \frac{\partial V_{\text{eff}}}{\partial v} = v \left[ \frac{\lambda}{2} v^2 - \mu^2 + \frac{\alpha^2}{R^2} \right], \quad (8)$$

$$0 = \frac{\partial V_{\text{eff}}}{\partial \alpha} = \alpha \left[ \frac{v^2}{R^2} - \frac{N_f}{3\pi^2 R^4} (2\alpha^2 - 3\alpha + 1) \right]. \quad (9)$$

These equations lead to the following four types of solutions as possible vacuum configurations:

(I) type I

$$\begin{cases} \alpha_{\text{I}} = \frac{1}{2}, \\ v_{\text{I}} = 0, \end{cases} \quad (10)$$

(II) type II

$$\begin{cases} \alpha_{\text{II}} = 0, \\ v_{\text{II}} = \sqrt{\frac{2}{\bar{\lambda}}} \mu, \end{cases} \quad (11)$$

(III) type III<sub>±</sub>

$$\begin{cases} \alpha_{\text{III}\pm} = \frac{3\bar{\lambda} \pm \sqrt{4(1+2\bar{\lambda})(R\mu)^2 - \bar{\lambda}(4-\bar{\lambda})}}{2(1+2\bar{\lambda})}, \\ v_{\text{III}\pm} = \sqrt{\frac{2}{\bar{\lambda}}} \left( \mu^2 - \frac{\alpha_{\text{III}\pm}^2}{R^2} \right), \end{cases} \quad (12)$$

(IV) type IV

$$\begin{cases} \alpha_{\text{IV}} = 0, \\ v_{\text{IV}} = 0, \end{cases} \quad (13)$$

where

$$\bar{\lambda} \equiv \frac{N_f \lambda}{6\pi^2}. \quad (14)$$

Let us first clarify the gauge symmetry breaking for each configuration. For the type I solution, the gauge field for the  $S^1$  direction acquires a nonvanishing vacuum expectation

value, although it does not lead to any symmetry breaking, because the Wilson line

$$\exp \left\{ ig \int_0^{2\pi R} dy \langle A_y \rangle \right\} = \exp \left\{ i \begin{pmatrix} \pi & 0 \\ 0 & -\pi \end{pmatrix} \right\} = -\mathbf{1} \quad (15)$$

is proportional to the identity matrix[8]. For the type II and III solutions, the  $SU(2)$  gauge symmetry is completely broken. For the type IV solution, the  $SU(2)$  gauge symmetry is unbroken. We refer to the phases of the type I, II and III solutions as the Hosotani, Higgs and coexisting phases, respectively.

To determine which solution gives the minimum energy, we first evaluate the effective potential for the type I, II, IV solutions and study the stability against small fluctuations around the type I, II, III solutions. Since  $V_{\text{eff}}(\alpha_I, v_I; R) < V_{\text{eff}}(\alpha_{IV}, v_{IV}; R)$  for any  $R$ , the type IV solution is not the vacuum configuration. Since  $V_{\text{eff}}(\alpha_I, v_I; R) > V_{\text{eff}}(\alpha_{II}, v_{II}; R)$  when  $R > R_1$  and  $V_{\text{eff}}(\alpha_I, v_I; R) < V_{\text{eff}}(\alpha_{II}, v_{II}; R)$  when  $R < R_1$ , the type I (II) solution is not the vacuum configuration when  $R > R_1$  ( $R < R_1$ ), where  $R_1$  is given by

$$R_1 = \left( \frac{\bar{\lambda}}{8} \right)^{\frac{1}{4}} \frac{1}{\mu}. \quad (16)$$

The stability arguments against small fluctuations show that the type I (II) solution becomes unstable when  $R > R_2$  ( $R < R_3$ ) and that the type III<sub>+</sub> (III<sub>-</sub>) solution is always unstable (locally stable), where  $R_2$  and  $R_3$  are given by

$$R_2 = \frac{1}{2\mu}, \quad (17)$$

$$R_3 = \frac{\sqrt{\bar{\lambda}}}{\mu}. \quad (18)$$

It should be noted that the vacuum expectation value  $\alpha$  is physically equivalent to  $\alpha'$  if  $\alpha' - \alpha \in \mathbf{Z}$ . This is because  $\langle A_y \rangle$  itself is not a direct physical observable, but the Wilson line  $e^{i2\pi g \langle A_y \rangle R}$  for the  $S^1$  direction is. This fact and the symmetry of the effective potential under  $\alpha \rightarrow -\alpha$  allow us to restrict our consideration to the range

$$0 \leq \alpha \leq \frac{1}{2} \quad (19)$$

without loss of generality. Then, for the type III<sub>-</sub> solution,  $\alpha_{\text{III-}}$ , to lie in the above region, the radius of  $S^1$  is restricted to the range  $R_4 \leq R \leq R_3$  for  $\bar{\lambda} \leq 1$  and  $R_2 \leq R \leq R_3$  for  $\bar{\lambda} > 1$ , where  $R_4 = \sqrt{\bar{\lambda}(4 - \bar{\lambda})}/4(1 + 2\bar{\lambda})\mu^{-1}$ .

The relative magnitudes of  $R_1, \dots, R_4$  depend on  $\bar{\lambda}$ . It is not difficult to show that  $R_4 < R_3 < R_1 < R_2$  for  $\bar{\lambda} < \frac{1}{8}$ ,  $R_4 < R_1 < R_3 < R_2$  for  $\frac{1}{8} < \bar{\lambda} < \frac{1}{4}$ ,  $R_4 < R_1 < R_2 < R_3$  for  $\frac{1}{4} < \bar{\lambda} < \frac{1}{2}$  and  $R_4 < R_2 < R_1 < R_3$  for  $\bar{\lambda} > \frac{1}{2}$ .<sup>6</sup> It turns out that the three parameter regions i)  $\bar{\lambda} < \frac{1}{8}$ , ii)  $\frac{1}{8} < \bar{\lambda} < 1$  and iii)  $\bar{\lambda} > 1$  lead to different phase structures with respect to  $R$ . We study each region separately below.

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<sup>6</sup>Note that  $R_4$  is defined only for  $\bar{\lambda} \leq 4$ .

i)  $\bar{\lambda} < \frac{1}{8}$

It follows from the above analyses of the potential energy and the stability that the vacuum configuration can uniquely be determined, except in the region  $R_4 < R < R_3$ , in which there are two possibilities, type I and III<sub>-</sub>, of the vacuum. Comparing the effective potentials for the type I and III<sub>-</sub> solutions directly, we can show that  $V_{\text{eff}}(\alpha_{\text{I}}, v_{\text{I}}; R) < V_{\text{eff}}(\alpha_{\text{III-}}, v_{\text{III-}}; R)$  for  $R_4 \leq R \leq R_3$ . This fact is sufficient to determine the vacuum configuration in the entire range of  $R$ . The result is

$$(\alpha, v) = \begin{cases} (\alpha_{\text{I}}, v_{\text{I}}) & \text{for } R < R_1, \\ (\alpha_{\text{II}}, v_{\text{II}}) & \text{for } R > R_1. \end{cases} \quad (20)$$

Since the type I solution is not continuously connected to the type II solution at  $R = R_1$ , a first-order phase transition occurs there.

ii)  $\frac{1}{8} < \bar{\lambda} < 1$

It follows from the analyses given previously in this section that the vacuum configuration can uniquely be determined, except in the region  $R_4 < R < R_1$  ( $R_4 < R < R_2$ ) with  $\frac{1}{8} < \bar{\lambda} < \frac{1}{2}$  ( $\frac{1}{2} < \bar{\lambda} < 1$ ), in which there are two possibilities, the type I and III<sub>-</sub>, of the vacuum. Comparing the effective potentials for the type I and III<sub>-</sub> solutions directly, we can show that  $V_{\text{eff}}(\alpha_{\text{I}}, v_{\text{I}}; R) < V_{\text{eff}}(\alpha_{\text{III-}}, v_{\text{III-}}; R)$  for  $R_4 < R < R_5$  and that  $V_{\text{eff}}(\alpha_{\text{I}}, v_{\text{I}}; R) > V_{\text{eff}}(\alpha_{\text{III-}}, v_{\text{III-}}; R)$  for  $R_5 < R < R_1$  ( $R_5 < R < R_2$ ) with  $\frac{1}{8} < \bar{\lambda} < \frac{1}{2}$  ( $\frac{1}{2} < \bar{\lambda} < 1$ ), where  $R_5$  is given by

$$R_5 = \frac{1}{2\mu} \sqrt{\frac{3 + 8\bar{\lambda} - 2(1 + 2\bar{\lambda})\sqrt{2(1 + 2\bar{\lambda})}}{1 + 2\bar{\lambda}}}. \quad (21)$$

Thus, we can conclude that the vacuum configuration is given by

$$(\alpha, v) = \begin{cases} (\alpha_{\text{I}}, v_{\text{I}}) & \text{for } R < R_5, \\ (\alpha_{\text{III-}}, v_{\text{III-}}) & \text{for } R_5 < R < R_3, \\ (\alpha_{\text{II}}, v_{\text{II}}) & \text{for } R > R_3. \end{cases} \quad (22)$$

Since the type I solution is not continuously connected to the type III<sub>-</sub> solution at  $R = R_5$ , a first-order phase transition occurs there. Since the type III<sub>-</sub> solution becomes identical to the type II solution at  $R = R_3$ , the phase transition at  $R = R_3$  is second order.

iii)  $\bar{\lambda} > 1$

In this case, the analyses given previously in this section turn out to be sufficient to determine the vacuum configuration uniquely. The result is

$$(\alpha, v) = \begin{cases} (\alpha_{\text{I}}, v_{\text{I}}) & \text{for } R < R_2, \\ (\alpha_{\text{III-}}, v_{\text{III-}}) & \text{for } R_2 < R < R_3, \\ (\alpha_{\text{II}}, v_{\text{II}}) & \text{for } R > R_3. \end{cases} \quad (23)$$



Since the vacuum configuration is found to be connected continuously at the critical radii  $R = R_2$  and  $R_3$ , the phase transitions are both second order. All the results obtained above are summarized in Fig. 1.

It is instructive to clarify the reason why different phases appear for small  $R$  and large  $R$ . For  $R \ll \mu^{-1}$ , the Higgs potential  $V(\phi)$  becomes irrelevant, and the leading contribution to the effective potential comes from the radiative correction  $\Delta V(\alpha; R)$ , so that  $\alpha$  is given by the configuration that minimizes  $\Delta V(\alpha; R)$ , *i.e.*  $\alpha = \frac{1}{2}$ . The next-to-leading term is the third term in the effective potential (7), and it forces  $v$  to vanish with  $\alpha \neq 0$ . Thus, the Hosotani phase with  $\alpha = \frac{1}{2}$  and  $v = 0$  is expected to be realized for  $R \ll \mu^{-1}$ . This is consistent with our results obtained before. For  $R \gg \mu^{-1}$ , the radiative correction  $\Delta V(\alpha; R)$  becomes irrelevant, and the leading contribution to the effective potential comes from the Higgs potential  $V(\phi)$ , so that  $v$  is given by the configuration that minimizes  $V(\phi)$ , *i.e.*  $v = \sqrt{\frac{2}{\lambda}}\mu$ . The next-to-leading term is the third term in the effective potential (7), and it forces  $\alpha$  to vanish with  $v \neq 0$ . Thus, the Higgs phase with  $\alpha = 0$  and  $v = \sqrt{\frac{2}{\lambda}}\mu$  is expected to be realized for  $R \gg \mu^{-1}$ . This is also consistent with our results obtained before.

The above arguments concerning the size of  $R$  can be applied to any gauge theory with Higgs fields; the Hosotani mechanism plays an important role in determining the phase structure for small  $R$ , while the Higgs mechanism plays an important role for large  $R$ . Therefore, different symmetry breaking mechanisms work for small and large  $R$ . This is the reason for the occurrence of phase transitions. Since the Hosotani mechanism, in general, works on gauge theories on non-simply connected spaces,[8] the phase structure found in the  $SU(2)$  gauge model on  $M^3 \otimes S^1$  is expected to be a general feature of gauge theories with Higgs fields on non-simply connected spaces.

Before closing this section, we should make a few comments on the determination of phase structures. Strictly speaking, the argument for small (large)  $R$  given above holds only for  $R \rightarrow 0$  ( $R \rightarrow \infty$ ), in general, and does not necessarily imply the existence of a Hosotani (Higgs) phase for small (large) but *finite*  $R$ . In fact, for instance, an  $SU(4)$  gauge model with the same matter content as the  $SU(2)$  model has no Higgs phase[13]. Furthermore, the arguments for small and large  $R$  provide no information regarding a coexisting phase. To determine phase structures precisely, we must study the whole structure of effective potentials more carefully.

## 4 A GUT Scenario

In this section, we propose a GUT scenario to clarify some of the phenomenological implications of our results and discuss how the hierarchy problem is reinterpreted in our scenario.

As an illustration, let us consider an  $SU(5)$  GUT model on  $M^4 \otimes S^1$ , in which a Higgs field belongs to the fundamental representation and all mass scales are set to a GUT scale  $M_G$ .

Suppose that the  $SU(5)$  gauge symmetry is broken to  $SU(3) \times SU(2) \times U(1)$  by the Hosotani mechanism with a nontrivial Wilson line for small  $R$ <sup>7</sup> and that the Higgs field breaks  $SU(3) \times SU(2) \times U(1)$  to  $SU(3) \times U(1)_{\text{em}}$  for large  $R$ . Then, a phase transition occurs at some critical radius  $R_*$  above which  $SU(2) \times U(1)$  is broken to  $U(1)_{\text{em}}$ . This critical radius  $R_*$  is of order  $M_G^{-1}$ , which is the unique mass scale of the model. If the radius  $R$  of  $S^1$  stays just above the critical radius  $R_*$  and the phase transition at  $R = R_*$  is second order, the breaking scale of  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$  is of order  $R^{-1} \sim R_*^{-1} \sim M_G$ , while the breaking scale of  $SU(2) \times U(1) \rightarrow U(1)_{\text{em}}$  can be much smaller than  $M_G$ . This is because  $SU(2) \times U(1)$  is unbroken at  $R = R_*$ , and hence the breaking scale is expected to be very small.

Our GUT scenario has some advantages. No Higgs field belonging to the adjoint representation is necessary, because the  $S^1$  component of gauge fields plays its role. The hierarchy problem is replaced in our scenario by the question of why the radius of  $S^1$  is so close to the critical radius. Although our scenario does not solve the fine-tuning problem, it allows us to reinterpret the hierarchy problem as a dynamical problem to determine the radius of  $S^1$ . By minimizing the potential with respect to  $R$ , we could, in principle, determine the “expectation value” of the radius  $R$ [12]. Our scenario might be stable against quantum corrections, so that supersymmetry might not be necessary. It is of great interest to seek mechanisms to stabilize the radius in the vicinity of the critical radius.

## 5 Conclusions and Discussions

We have investigated an  $SU(2)$  gauge model with a Higgs field on  $M^3 \otimes S^1$  and shown that the model has three phases: Hosotani, coexisting and Higgs phases. We have also obtained the critical radius and determined the order of the phase transitions, as depicted in Fig. 1. The non-triviality of the phase structure is not, however, peculiar to this model but, rather, is expected to be a general feature of gauge theories with Higgs fields on non-simply connected spaces. Phase structures of such theories, in general, depend on matter content as well as gauge groups. A variety of phase diagrams appear. Actually, if we replace the fermions in the fundamental representation by those in the adjoint representation in our  $SU(2)$  gauge model, we obtain a phase diagram similar to Fig. 1, but in this case the  $SU(2)$  gauge symmetry is broken to  $U(1)$  in the Hosotani phase. If we replace the Higgs

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<sup>7</sup> This symmetry breaking could be realized by choosing fermion matter content and/or the boundary conditions appropriately[11].

field in the fundamental representation by that in the adjoint representation, the phase diagram becomes trivial.<sup>8</sup>

In the analysis of Section 3, we ignored contributions from the gauge, ghost and Higgs one-loop diagrams by taking  $N_f$  to be large. For small  $N_f$ , they contribute to the effective potential, so that the phase structure for small  $R$  is more complicated. A further complication arises when mass terms are added to the fermions. Then, quantum corrections from massive fermions with mass  $m_f$  survive for small  $R$  but are exponentially suppressed for  $R \gg m_f^{-1}$ . When constructing phenomenological models, we should take them into account correctly.

Other straightforward extensions of the  $SU(2)$  gauge model consist of replacing the  $SU(2)$  gauge group by  $SU(N)$  and the space-time  $M^3 \otimes S^1$  by  $M^D \otimes S^1$ . The latter extension will not drastically alter qualitative features, because the  $R$  dependence of radiative corrections is not sensitive to the dimensionality of the space-time. It turns out that the  $SU(N)$  models with  $N$  even ( $N \geq 4$ ) have only two phases without a Higgs phase, whereas the  $SU(N)$  models with  $N$  odd have phase structures similar to that of the  $SU(2)$  gauge model. If fermions in various representations are added, the analysis becomes more involved. The details of these extensions will be reported elsewhere. [13]

As a phenomenological application of our discovery of phase structures in gauge theories, we have proposed a GUT scenario in which the gauge hierarchy problem is replaced by the dynamical problem of the stabilization of the radius of the compactified space in the vicinity of a critical radius. To construct a realistic GUT model, it is necessary to find models with desired symmetry breaking patterns accompanied by phase transitions and explore the possibility of stabilizing the radius in the vicinity of a critical radius. This might be worth investigating.

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<sup>8</sup>This result comes from the fact that the interaction term between the gauge and the Higgs fields vanishes, because the vacuum expectation values of both fields are diagonal.

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